

# Three-Dimensionally Coupled Microstrip Lines via a Rotated Slot in a Common Ground Plane

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## ABSTRACT

Microstrip lines coupled through a slot in a common ground plane are analyzed. The slot is rotated with respect to the propagating axis of the microstrips. The analysis is based on the spectral domain method and the reciprocity theorem. Numerical results of the S parameters are presented for different values of angle of rotation of the slot.

## INTRODUCTION

In this paper, we analyze the problem of two microstrip lines coupled by a slot in the common ground plane that is rotated from the axis of microstrips. (See Figs.1 and 2.). This problem is of interest for MIC and MMIC design. Although a similar problem with a small rectangular slot oriented perpendicular to the microstrip axis has been reported [1], no results with a rotated slot seem to be available. The information on the structure with such a rotated slot is believed to increase design flexibility.

The method of analysis in this paper proceeds in the following manner. The method is essentially based on the one reported in [1] which is extended here for more general cases. It is still assumed that the slot is very narrow so that the electric field is only in the direction of the gap. The electric field in the slot is assumed to be given by a single piecewise sinusoidal (PWS) function with an unknown amplitude. However, unlike [1], the current on the microstrip is considered to be consisting of the axial and transverse components[2],[3]. We consider the dominant mode incidence from Port 1 in Fig.1. Infinitely many modes are excited by the slot discontinuity. By means of the reciprocity theorem and of the boundary condition

in the slot, the unknown amplitude of the electric field in the slot is found. From this quantity, all the scattered field coefficients can be found. S parameters needed for the circuit design are found from the dominant mode field coefficients. For the special case in which the slot is oriented perpendicular to the microstrip axis, the results agree with those reported by Pozar[1].

## FORMULATION OF THE PROBLEM

Fig.1 shows a slot coupled microstrip lines. A slot is located on the common ground plane of the microstrip lines, and is rotated with an angle as shown in Fig. 2. The slot field is represented by a PWS function with unknown amplitude[4] in terms of the local coordinates  $(\xi, \zeta)$ .

$$\mathbf{E}^s = E_0 e^s(\xi) \mathbf{a}_\zeta \quad (1)$$

where

$$e^s(\xi) = \begin{cases} \frac{\sin k_e(L/2 - |\xi|)}{W \sin(k_e L/2)} & |\xi| \leq \frac{L}{2}, |\zeta| \leq \frac{W}{2} \\ 0 & \text{elsewhere} \end{cases} \quad (2)$$

The microstrip lines are assumed to be infinitely long. The total magnetic field of the microstrip lines I and II in the two regions  $z \leq z_1$  and  $z \geq z_2$  in Fig.1 can be written as

$$\begin{aligned} \mathbf{H}_1 &= A_1 \mathbf{H}_{11}^+ + R A_1 \mathbf{H}_{11}^- + \sum_n A_n \mathbf{H}_{1n}^- & z \leq z_1 \\ &= T A_1 \mathbf{H}_{11}^+ + \sum_n B_n \mathbf{H}_{1n}^+ & z \geq z_2 \end{aligned} \quad (3)$$

and

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$$\begin{aligned} \mathbf{H}_2 &= \sum_n \mathbf{C}_n \mathbf{H}_{2n} & z \leq z_1 \\ &= \sum_n \mathbf{D}_n \mathbf{H}_{2n}^+ & z \geq z_2 \end{aligned} \quad (4)$$

respectively [5], where the prime on the summation means exclusion of the dominant mode. Also, R and T are reflection and transmission coefficients, respectively. Other coefficients are unknown amplitude constants. The total electric fields can be given in a form similar to the magnetic fields. Applying the reciprocity theorem to the structure, relationships between these coefficients and  $E_0$  are obtained.

It is assumed that these relationships are valid for the region between  $z_1$  and  $z_2$  [6]. In the slot at  $y = 0$  and  $\zeta = 0$ , the boundary condition is

$$\mathbf{a}_\xi \cdot (\mathbf{H}_1 + \mathbf{H}_1^s) = \mathbf{a}_\xi \cdot (\mathbf{H}_2 + \mathbf{H}_2^s) \quad (5)$$

In Eq.(5)  $\mathbf{H}_1^s$  and  $\mathbf{H}_2^s$  are given by

$$\mathbf{H}_1^s = \int_S \mathbf{G}_1 \cdot \mathbf{M}_1 da \quad (6)$$

$$\mathbf{H}_2^s = \int_S \mathbf{G}_2 \cdot \mathbf{M}_2 da \quad (7)$$

where  $\mathbf{G}_1$  and  $\mathbf{G}_2$  are dyadic Green functions[7].  $\mathbf{M}_1$  and  $\mathbf{M}_2$  are the magnetic currents at  $y = 0^+$  and  $y = 0^-$  in the slot, respectively, and S indicates the slot surface.

If the dominant mode is propagating in the microstrip lines, S parameters are given by

$$\begin{aligned} S_{11} &= R \\ S_{21} &= T \\ S_{31} &= D_1 \sqrt{\frac{Z_{c1}}{Z_{c2}}} \end{aligned} \quad (8)$$

where  $Z_{c1}$  and  $Z_{c2}$  are the characteristic impedances of the microstrip lines I and II, respectively.

## NUMERICAL RESULTS

Numerical results of  $S_{11}$ ,  $S_{21}$  and  $S_{31}$  for different

angles of rotation are shown in Figs. 3(a), (b) and (c), respectively. In these figures the results computed and measured for  $\theta = 0$  by Pozar are also shown for comparison. It is found that the results by the present method agree well with Pozar's results[1]. The computed results show that  $S_{11}$  and  $S_{31}$  decrease while  $S_{21}$  increase with the angle of the rotation. This tendency indicates that the coupling between the upper and lower microstrip lines decreases as the slot is rotated. It is interesting to compare the present results with those for the rotated slot in a broad wall of a waveguide[8]. It was reported in [8] that the slot admittance increases as the rotation angle is increased, resulting in the decrease of the reflection coefficient. Such a situation is similar to the present results.

## CONCLUSIONS

We have analyzed the coupling problem of two microstrip lines through the small rectangular rotated slot. The method is based on the reciprocity theorem. The results obtained are believed useful for the circuit design in MIC and MMIC.

## REFERENCES

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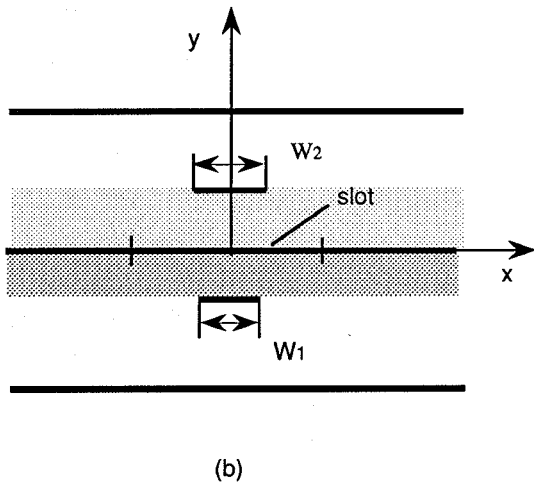
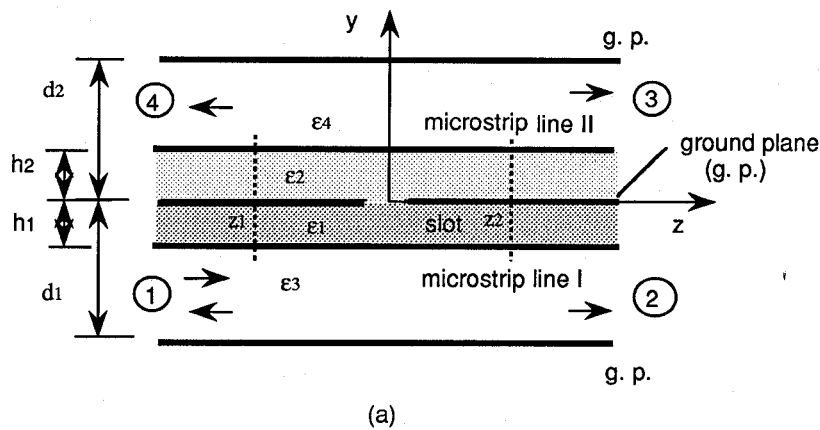


Fig. 1 Geometry of the slot coupled microstrip lines.

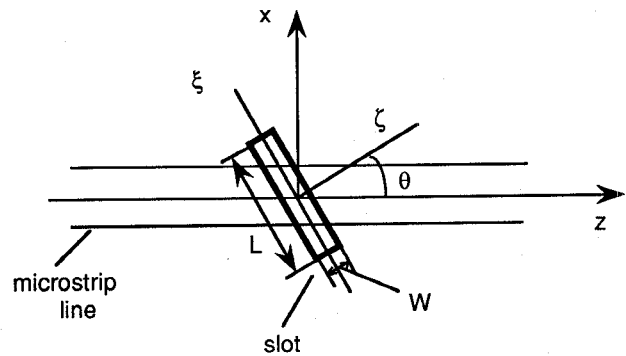
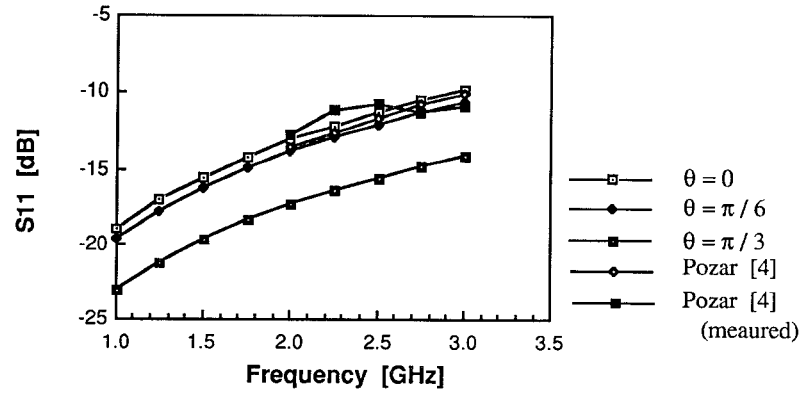
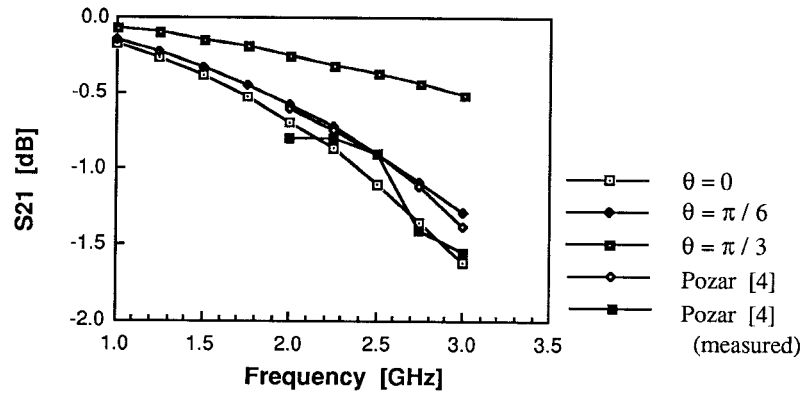


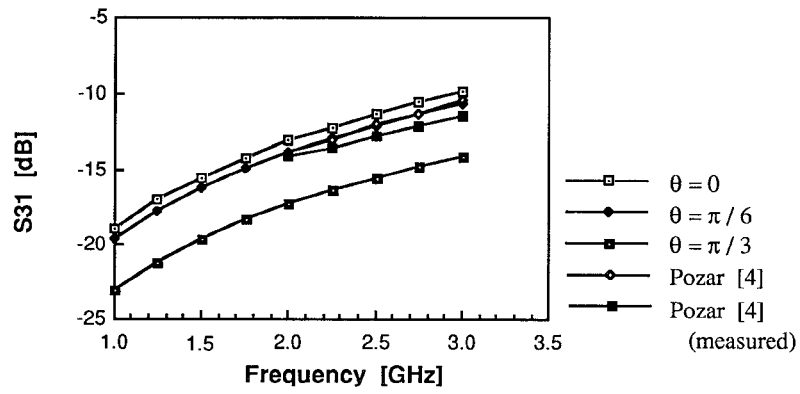
Fig. 2 Rotated Slot.



(a)



(b)



(c)

Fig. 3 Numerical results of the Sparameters  $S_{11}$ ,  $S_{21}$  and  $S_{31}$ .

$W_1 = W_2 = 2.54$  mm,  $h_1 = h_2 = 0.762$  mm,  $d_1 = d_2 = 50$  mm

$W = 1.1$  mm,  $L = 15$  mm,  $\epsilon_1 = \epsilon_2 = 2.22$ , and  $\epsilon_3 = \epsilon_4 = 1$ .